Journal of Computational Physics 228 (2009) 4965–4969

Contents lists available at ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

Short Note

A modified artificial viscosity approach for compressible turbulence simulations

Ankit Bhagatwala [a,*], Sanjiva K. Lele [a,b]

[a] Department of Aeronautics and Astronautics, 496 Lomita Mall, Stanford, CA 94305, USA
[b] Department of Mechanical Engineering, Stanford University, Stanford, CA 94305, USA

ARTICLE INFO

Article history:
Received 15 December 2008
Accepted 8 April 2009
Available online 19 April 2009

Keywords:
Shocks
Turbulence
Large-eddy simulation
Hyperviscosity

1. Introduction

Standard high order methods give rise to spurious oscillations near shocks which can be controlled by using localized artificial viscosity (AV) with high wavenumber bias. Using simulations of compressible isotropic turbulence with optimized high-order schemes at different resolutions we investigated the range of scales where artificial dissipation is active. We observed that the impact of AV was not limited to high wavenumbers. This is especially true for moderately high Mach number isotropic turbulence which spontaneously forms shocklets, for which the AV method is found to excessively damp the dilatational motions. We propose a modified form using a modified coefficient which activates AV only in the regions of strong compression, such as shocks, turning it completely off for turbulence and expansion waves. This is found to give improved statistics for all quantities, not just dilatation. This formulation reverts back to the traditional one for strong shocks, so that its shock capturing capability is not compromised (see Fig. 1).

2. Formulation

The non-dimensional Navier–Stokes equations can be written as

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j + p\delta_{ij}) = \frac{\partial \tau_{ij}}{\partial x_j}, \tag{2}$$

* Corresponding author.
E-mail addresses: ankitb@stanford.edu, ankitvb@gmail.com (A. Bhagatwala).

doi:10.1016/j.jcp.2009.04.009

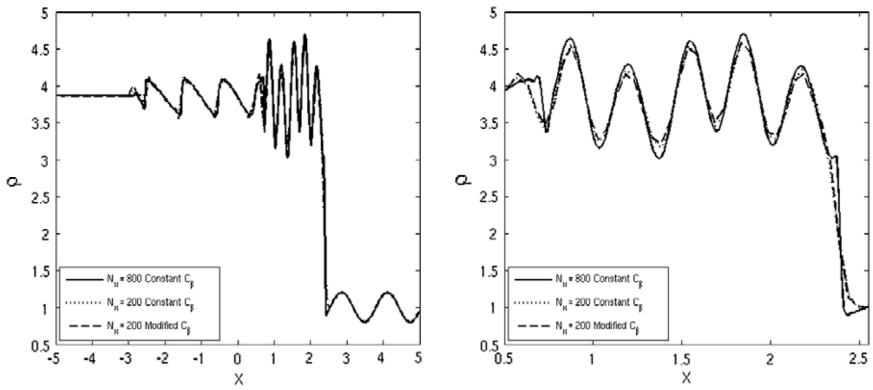

Fig. 1. Left: Comparison between the modified and constant coefficient bulk viscosity for Shu–Osher problem with $N_x = 200$ at t = 1.8. Right: Zoomed in comparison for post-shock entropy waves.

$$\frac{\partial}{\partial t}(\rho e_t) + \frac{\partial}{\partial x_j}((\rho e_t + p)u_j) = -\frac{\partial q_j}{\partial x_j} + \frac{\partial}{\partial x_j}(u_i \tau_{ij}). \tag{3}$$

The viscous stress τ_{ij} and the heat flux q_i are

$$\tau_{ij} = \frac{\mu}{\mathrm{Re}_\infty}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \frac{\mu}{\mathrm{Re}_\infty}\left(\beta - \frac{2}{3}\right)\frac{\partial u_k}{\partial x_k}\delta_{ij}, \tag{4}$$

$$q_i = -\frac{\mu}{\mathrm{Re}_\infty \mathrm{Pr}}\frac{\partial T}{\partial x_i}, \tag{5}$$

where the Reynolds and Prandtl numbers are $\mathrm{Re}_\infty = \rho^*_\infty L^* a^*_\infty / \mu^*_\infty$ and $\mathrm{Pr} = c^*_p \mu^* / \kappa^*$. Following Cook [1], artificial terms are now added to the physical viscosity coefficients. $\mu = \mu_f + \mu_h$, $\beta = \beta_f + \beta_h$ and $\kappa = \kappa_f + \kappa_h$. The artificial terms take the form

$$\mu_h = C_\mu \overline{\rho|\nabla^4 S|}\Delta^6, \quad \beta_h = C_\beta \overline{\rho|\nabla^4 S|}\Delta^6, \quad \kappa_h = C_\kappa \overline{\frac{\rho c}{T}|\nabla^4 e|}\Delta^5, \tag{6}$$

where $C_\mu = 0.002$, $C_\beta = 1.0$ and $C_\kappa = 0.01$.

3. Modified artificial viscosity

In the preceding section the artificial viscosity can be considered a constant coefficient formulation. The double Laplacian in (6) gives a high wavenumber bias to the dissipation introduced by this scheme. However in simulations of underresolved isotropic turbulence, this dissipation is found to excessively damp the dilatational modes and also pressure, density and temperature fluctuations as shown in Figs. 2 and 3. Hence a modified formulation is considered which imparts a further selectivity to the artificial dissipation. The *modified* refers to the fact that the coefficient C_β is now calculated at every grid point based on the following formula:

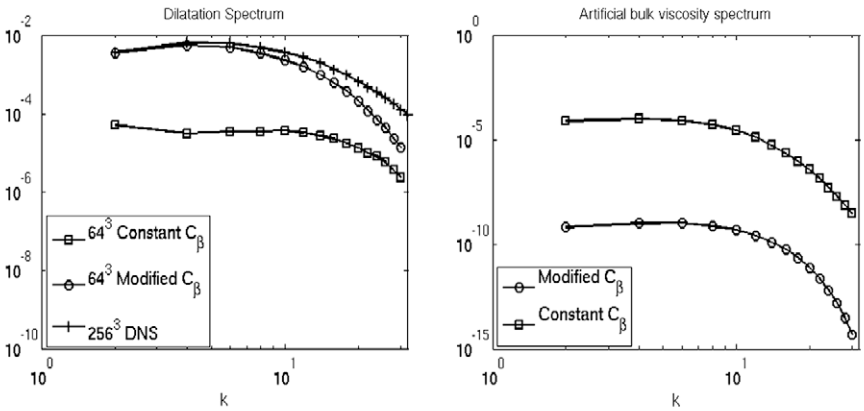

Fig. 2. Comparison of Dilatation and artificial bulk viscosity spectra for 64^3 isotropic turbulence. Plus: 256^3 DNS. Square: constant coefficient model. Circle: modified coefficient model.

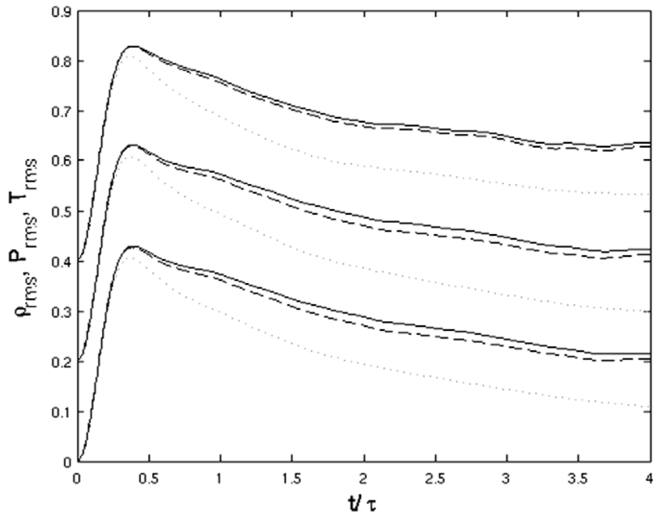

Fig. 3. Comparison of RMS quantities for 64^3 isotropic turbulence. Solid line – 256^3 DNS, Dashed line – 64^3 modified coefficient model. Dotted line – 64^3 constant coefficient model.

$$C_\beta = \frac{1}{2}\left(1 - \tanh\left(2.5 + 10\frac{\Delta}{c}\nabla\cdot\mathbf{u}\right)\right) \times \frac{(\nabla\cdot\mathbf{u})^2}{(\nabla\cdot\mathbf{u})^2 + \Omega^2 + \epsilon}, \tag{7}$$

while C_μ and C_κ are unchanged and ϵ is a small number equal to 10^{-6}. The modified formulation employs two sensors the first of which is based on relative magnitudes of dilatation and vorticity following Ducros et al. [2]. The second sensor, which acts like a switch, turns the artificial bulk viscosity on or off based on the magnitude and sign of dilatation. In particular, for positive dilatation it is switched off whereas it rises slowly as the negative dilatation increases in magnitude. Since shocks are a region of high negative dilatation (strong compression waves), bulk viscosity needs to act only in such regions.

The factor $10\Delta/c$ in front of the dilatation is a scaling term. It makes the grid dependent numerical dilatation invariant to mesh size and also appropriately nondimensionalises the dilatation, irrespective of the specific nondimensionalisation used for the remaining equations. The factor $2\Delta/c$ corresponds to the time scale of the highest frequency acoustic wave that can be represented on the grid. This formulation starts damping dilatation only if the time period of the acoustic wave is less than 1/10th of the time scale of the dilatation (shock).

4. Results

4.1. Shu–Osher problem

The Shu–Osher problem is a one dimensional canonical shock-turbulence interaction in which a Mach 3 shock wave interacts with a sinusoidal density field [3]. Pressure is constant on either side of the shock, which gives an entropy wave downstream of the shock. The Euler equations are solved on a domain $x \in [-5, 5]$ with initial conditions

$$(\rho, u, p) = \begin{cases} (3.857143, 2.629369, 10.333333), & x < -4, \\ (1 + 0.2\sin(5x), 0, 1), & x \geqslant -4. \end{cases}$$

The goal here is to ensure that the shock capturing capability of the original scheme is preserved. The domain is discretized with $N = 200$ grid points. The solution is integrated to $t = 1.8$ and compared with the constant coefficient bulk viscosity and a converged reference solution as shown in figures. The amplitude of the post shock entropy waves, position of the shock and position of the contact discontinuity separating the entropy and acoustic waves are all well captured just as with the constant coefficient bulk viscosity. This is clearly seen in Fig. 1.

4.2. Isotropic turbulence

High turbulence Mach number and high Reynolds number isotropic turbulence simulations were carried out. The Mach number was $M_t = 0.6$ and the Taylor scale Reynolds number was $Re_\lambda = 100$. This Mach number is high enough for the flow to spontaneously form shocklets and the Reynolds number is high enough to have a broad range of scales for vortical motions. It has been noted in other studies [4], that artificial bulk viscosity causes excessive dissipation of dilatational motions and gives erroneous RMS profiles for density, pressure and temperature. As seen in Figs. 3 and 4, these are significantly improved using the modified formulation.

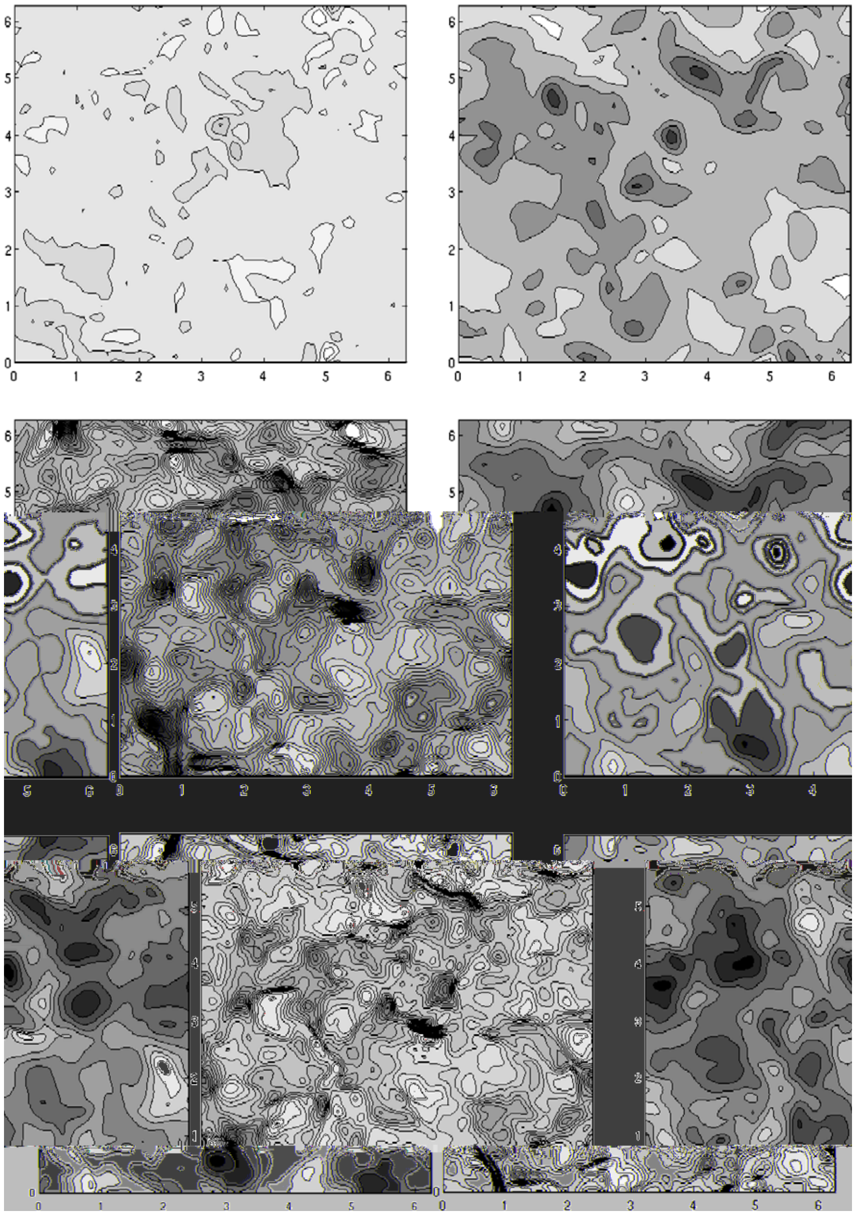

Fig. 4. Comparison of slices of dilatation and density contours: Dilatation on the left and density to the right. Top to bottom: constant C_β, 64^3 grid, Modified C_β, 64^3 grid and 256^3 grid DNS. Same contour and grayscale levels are used in both plots.

The formulation works by decreasing the magnitude of bulk artificial viscosity at *all scales* as seen in the plot of spectra in Fig. 2. Since the bulk viscosity acts on the dilatational motions only, it has no effect on vortical motions or overall turbulent kinetic energy. The improvement in prediction of the acoustic field, is quite dramatic, as can be seen in Figs. 2 and 4. The latter compares contours of dilatation and density for the two formulations. An eddy shocklet, which can be clearly seen in the figure on the right is completely obliterated by the constant coefficient formulation. Significant improvement is also seen in the thermodynamic quantities, density, temperature and pressure. This can be seen in the plot of RMS quantities (Fig. 3) and also in the density contours.

5. Conclusion

A modified formulation for artificial bulk viscosity is proposed which retains the shock capturing capability of the original formulation while significantly improving performance in terms of acoustic motions and thermodynamic fluctuations for high Mach number turbulence. While this formulation works well for shock waves, it needs to be integrated with a more complete subgrid scale model for vortical motions as well. Further validation is needed for more complex shock turbulence problems. This is the focus of ongoing research.

Acknowledgments

This work was supported by DoE under the SciDAC project Grant Number DE-FC02-06-ER25787. The authors wish to thank Dr. Johan Larsson for many helpful discussions.

References

[1] A.W. Cook, Artificial fluid properties for large-eddy simulation of compressible turbulent mixing, Phys. Fluids (2007).
[2] F. Ducros, V. Ferrand, F. Nicoud, C. Weber, D. Darracq, C. Gacherieu, T. Poinsot, Large-eddy simulation of the shock/turbulence interaction, J. Comput. Phys. (1999).
[3] C.W. Shu, S. Osher, Efficient implementation of essentially non-oscillatory shock capturing schemes, J. Comput. Phys. (1989).
[4] E. Johnsen, J. Larsson, A.V. Bhagatwala, W.H. Cabot, P. Moin, B.J. Olson, P.S. Rawat, S.K. Shankar, B. Sjogreen, H.C. Yee, X. Zhong, S.K. Lele, Assessment of high-resolution methods for numerical simulations of compressible turbulence with shock waves. J. Comput. Phys., submitted for publication.